Modification of Vogel's Approximation Method for Optimality of Transportation Problem by Statistical Technique

Sabeen Sahito[1,*], Wajid A. Shaikh[2], A. G. Shaikh[3], Asif Ali Shaikh[1], Syed Feroz Shah[1]

[1]Department of Basic Sciences & Related Studies, MUET, Jamshoro, Sindh, Pakistan
[2]Department of Mathematics & Statistics, QUEST, Nawabshah, Pakistan
[3]Department of Basic Sciences & Related Studies, QUEST, Nawabshah, Pakistan
[*]Corresponding author: sabeensahito2@gmail.com

Abstract

This paper addresses the Optimality of Transportation Problems (TP). The modification in Vogel's Approximation Method for optimality of transportation problem by statistical technique (MVOTPST) is proposed, which provides the better feasible solution in comparison to the existing methods such as North West Corner Method (NWCM), Least Cost Method (LCM) and Vogel's Approximation Method (VAM). Various examples have been examined from the literature to validate the authenticity and optimality of the proposed MVOTPST method. Furthermore, the Initial Basic Feasible Solution (IBFS) of the modified technique has been found either absolutely optimal, close to optimal, same as VAM, or better than the existing methods.

Keywords—Optimal Solution; Transportation Problems; Modified Vogel's Approximation Method; Statistical Technique; Initial Basic Feasible Solution.

1 Introduction

Operations research (OR)is an investigative and rational approach in the field of Mathematics for directing, governing, problem-solving and decision-making that is useful in addressing the problems in the management of organizations, businesses, government and society. It is a set of intensively applicable data-organizing and decision-making strategies that are highly useful in all kinds of Business, Trade, Managerial Organizations and Transport. Furthermore, it is a field of operations regarding assessing abstract thinking and the best choice of decisions to solve Linear and non-linear programming problems. Linear Programming is a cost-minimizing or maximizing technique of a function under the dependence of constraints. Transportation Problem, a renowned Linear Programming Application, is used in this field to reduce the overall transportation cost between Supply and Demand (origin and destination). The word Transport needs no illustration for getting to understand the concept of the TP. Many approaches are used in order to minimize the Transportation cost, but the most efficient methods are the NWCM, the LCM, and VAM for IBFS. In addition, Modified Distribution Method (MODI) and the Stepping Stone Method (SSM) can be utilized to confirm that a TP's optimal solution is achieved.

The mathematical representation of the transportation problem is given as follows.

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} V_{ij}X_{ij} \quad (1)$$

where V_{ij} and X_{ij} represent the costs and allocations, respectively.

$$\sum_{j=1}^{n} X_{ij} \leq S_i,\ i = 1, 2, 3, \ldots, m \quad (2)$$

$$\sum_{i=1}^{m} X_{ij} \leq D_j,\ j = 1, 2, 3, \ldots, n \quad (3)$$

$$X_{ij} \geq 0, \quad \forall\, i, j$$

2 Existing & Proposed Methods

Operation Research involves different methods for solving and interpreting the outcomes of a TPs and

ISSN: 2523-0379 (Online), ISSN: 1605-8607 (Print)
DOI: https://doi.org/10.52584/QRJ.1902.07

their comparison with the optimal methods to measure the accuracy.

2.1 North West Corner MEthod

- The first step in addressing TP is to determine if it is balanced or not; if the problem is unbalanced, create a dummy row/column to balance it.
- Select the very first cell of the given table and assign as much value as possible.
- Cancel out demand or supply (row/column) that has been allocated to a cell to balance the TP again.
- Continue the process until both the demand and supply reach zero to obtain the appropriate IBFS.

2.2 Least Cost Method

- Select the cell having minimum cost/value in the TP table.
- Assign the maximum possible amount of demand or supply to that cell.
- Delete the corresponding row/column and continue from step 1 again.
- When all the demand and supply turn out to be zero, stop the process and find the minimum cost by multiplying every allocated cell by its cost

2.3 Vogel's Approximation Method

- If the specified transportation table is uneven, balance it; otherwise, initiate the operation.
- Select two lowest-cost cells in each row and column, and find penalty (difference).
- Once penalties have been calculated, select the penalty having maximum value, whether in demand or supply and allocate the value to the entry in the cell with the minimum cost in the corresponding row/column.
- Continue steps 2 and 3, until both the demand and supply become zero.

All three approaches are useful for TPs, but VAM is the most ideal since it improves the first basic feasible solution. Several researchers have presented modifications of the existing methods in their studies by developing different techniques to minimize the total transportation cost. However, some methods have given the same results as NWCM, LCM, VAM and some are still better at IBFS than these three methods.

Balakrishnan [1] has tackled the unbalanced TP by computing row and column penalties by disregarding the dummy row/column, which gives efficient IBFS. For tackling large-scale transportation problems. Korukoglu et al. [2] employed TOC in terms of allocation costs, the strategy considers the greatest three penalty costs. Hasan et al. [3] have analyzed that the ideal solution achieved by the SAM-Method and Zero Suffix Method creates a haphazard scenario in which someone is unsure if the answer is optimal or not. As a result, relying on their ideal answer cannot be prudent.

Babu et al. [4] have developed an unused scheme, LAM, which takes minimum iterations than existing methods and gives a solution nearer to optimality. Das et al. [5] have proposed a new strategy in order to overcome a few restrictions that show up when the most elevated cost penalty arises in two or more rows or columns. Das et al. [6] have found a computational error in VAM and have developed a new technique that gives a better lower doable solution than VAM which is close to the ideal solution. Babu et al. [7] have enhanced previous approaches and developed a new methodology in which the IBFS are less than VAM and near to optimality.

Junaid et al. [8] have minimized the transportation cost by computing the row-wise penalty between the two largest values, whereas the column-wise penalty is determined in each iteration using the two smallest values. Sood et al. [9] have developed a novel method for identifying the IBFS to a TP. The strategy described in this paper is known as the Maximum Difference Approach. This technique gives a simple, usable solution to the TP that is generally superior to VAM. Wali Ullah et al. [10] has modified VAM to provide a more efficient solution for a vast scale of TPs while avoiding many iterations. The author has estimated some distribution indicators. According to their findings, the technique gives the best possible solution.

Ahmed et al. [11] proposed a method (CVAM) for identifying IBFS for cost-effective TPs. They discovered that the customized strategy is simpler to compute and yields a better outcome. Samuel et al. [12] have developed a new Algorithmic approach to play down the uneven fuzzy TPs. Azad et al. [13] examined the averages of the discrepancies in cell values of each row and column from the lowest cell value of the corresponding row and column of the TT as penalties. This strategy yields ideal or close to optimal outcomes. Paul [14] has developed a revolutionary BFS strategy for reducing total transportation costs. Jamali et al. [15] have provided a summary of optimality of unused direct optimal methods for TPs.

Ali et al. [16] have used different means to minimize the total cost of balanced TPs. Khoso et al. [17], have developed an algorithm based on the Modified LCM's Approximation Method for decreasing the Transportation Cost entirely. Hussein et al., [18] have presented a modified technique of VAM, which gives better solutions almost nearer to optimality. The proposed strategy has a high chance of success in resolving TP. Karagul et al. [19] compared their method to six previously published preliminary solution strategies.

In comparison to the existing methodologies, the suggested method delivered the best initial solution in record time of these challenges. Finally, the suggested technique produced results as excellent as VAM and as speedy as the NWCM. The proposed method has improved the existing VAM by employing the Statistical technique. The motive behind the reframing of this method is to secure the IBFS to reach optimality. The proposed technique is applicable for both Balanced and Unbalanced TPs.

2.4 Algorithm of the Proposed (MVOTPST) Method

- Set supply and demand amounts of the problem. If the demand and supply amount is equal, then move to step 2; otherwise, balance the TP including a dummy row or column to make the demand and supply equal.

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} V_{ij} X_{ij}$$

- Compute the row penalty by selecting the two largest costs in a row and calculating their variance. Compute Column Penalty by selecting the two lowest costs in a column and calculating their variance

$$P = \delta^2 = \frac{\sum (V_{ij} - \bar{x})^2}{2} \tag{4}$$

where

$$\bar{x} = \frac{\sum V_{ij}}{2} \tag{5}$$

- Select the largest penalty in a row or column and allocate the smallest possible value $\min(S_i, D_j) = x_{ij}$ to the smallest cost of the associated row/column.
- In a well-balanced situation, if two penalties are equal, then select the row or column where there is minimum unit cost and minimum allocation as well in an unbalanced problem, choose the smallest element other than the zero elements of dummy row or column.
- Eliminate the entire row/column, where the demand or supply is zero.
- Stop the iterations when all the demand and supply become zero; otherwise, repeat step 2 until the feasible solution is obtained.

2.5 Solution of Transportation Problems by existing methods and the proposed method

The feasibility of the discussed TP models is examined first, and later the results are compared with results of existing methods. The TPs 1-5 (6-8,10) have been tested as case study by the existing methods and the proposed technique in order to assess the level of accuracy. Results of first TP 1 are presented with details, which given fair comparison. Later on, results of rest of problems are consolidated in single table against each problem.

2.5.1 *Transportation Problem*

TP 01 (4×5) is considered as in ref. [6] given in Table 2.

Sources/Destination	**D1**	**D2**	**D3**	**D4**	**Supply**
S1	6	8	10	9	**50**
S2	5	8	11	5	**75**
S3	6	9	12	5	**25**
Demand	**20**	**20**	**50**	**60**	**150**

TABLE 2: (4×5) TP 01

Solution of the TP 01 is determined by NWCM, LCM, VAM and the proposed MVOTPST methods, respectively, and described in Table 3-10.

	Destinations				**Supply**
Sources	6 (20)	8 (20)	10 (30)	9	**50**
	5	8	11 (40)	5 (35)	**75**
	6	9	12	5 (25)	**25**
Demand	**20**	**20**	**50**	**60**	**150**

TABLE 3: Solution of North West Corner Method (NWCM)

The total minimized cost is given as follows. $\mathbf{6 \times 20 + 8 \times 20 + 10 \times 10 + 40 \times 11 + 5 \times 35 + 5 \times 25 = 1120}$

	Destinations				**Supply**
Source	6	8	10 (50)	9	**50**
	5 (20)	8 (20)	11	5 (35)	**75**
	6	9	12	5 (25)	**25**
Demand	**20**	**20**	**50**	**60**	**150**

TABLE 4: Solution of Least Cost Method (LCM)

The minimized cost is as follows: $\mathbf{50 \times 10 + 20 \times 5 + 20 \times 8 + 35 \times 5 + 5 \times 25 = 1060}$

	Destinations				**Supply**
Sources	6 (20)	8	10 (30)	9	**50**
	5	8 (20)	11 (20)	5 (35)	**75**
	6	9	12	5 (25)	**25**
Demand	**20**	**20**	**50**	**60**	**150**

TABLE 5: Solution of Vogel's Approximation Method (VAM)

Sources/Destination	1	2	3	..	..	N	**Supply (Si)**
1	v_{11}	v_{12}	v_{13}	..	..	v_{1n}	s_1
2	v_{21}	v_{22}	v_{23}	..	..	v_{2n}	s_2
3	v_{31}	v_{32}	v_{33}	..	..	v_{3n}	s_3
.	.	.	.	..	..	.	.
.	.	.	.	..	..	.	.
m	v_{m1}	v_{m2}	v_{m3}			v_{mn}	s_i
Demand (Dj)	D_1	D_2	D_3			D_j	$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} D_j$

TABLE 1: General form of a transportation problem

The IBFS by VAM is as follows: $\mathbf{6 \times 20 + 10 \times 30 + 8 \times 20 + 11 \times 20 + 5 \times 35 + 5 \times 25 = 1100}$
Steps of the Solutions of the proposed MVOTPST method are given in Table 6-10.

	Destinations				Supply	P1
Sources	6	8	10	9	50	0.25
	5(20)	8	11	5	**75**	2.25
	6	9	12	5	25	2.25
Demand	**20**	20	50	60	=130	
P1	0.25	0	0.25	0		

TABLE 6: Proposed method step 1

	Destinations			Supply	P2
Sources	8	10	9	50	0.25
	8	11	5	55	2.25
	9	12	5 (25)	**25**	2.25
Demand	20	50	**60**	=105	
P2	0	0.25	0		

TABLE 7: Proposed method step 2

	Destinations			Supply	P3
Sources	8	10 (50)	9	50	0.25
	8 (20)	11	5	**55**	2.25
Demand	**20**	50	35	=85	
P3	0	0.25	0		

TABLE 8: Proposed method step 3

	Destinations		Supply	P4
Sources	10 (50)	9	50	0.25
	11	5 (35)	**35**	9
Demand	50	**35**	=50	
P4	0.25	4		

TABLE 9: Proposed method step 4

	Destinations	Supply	P4
Sources	10(50)	**50**	0.25
Demand	**50**	=0	
P4	0.25		

TABLE 10: Solution of the proposed method

METHODS	**IBF Solution**	**Optimal Solution**
MVOTPST	1060	1060
NWCM	1120	
LCM	1115	
VAM	1100	

TABLE 11: Comparison of the proposed model with the existing models for TP 01

The total shipping cost is given as follows: $10 \times 50 + 20 \times 5 + 20 \times 8 + 35 \times 5 + 25 \times 5 = 1060$. Table 11 shows that the proposed MVOTPST model has obtained the IBFS better than the existing models. Similarly, transportation problems 02-05 are solved by the existing and proposed models and the feasibility of the solutions is analysed and shown in Table 12.

2.5.2 Industrial Transportation Problem 06 [20]

Consider the (3×4) TP in Table 13 for different locations, as well as their corresponding Toyota firm sources. Each location's demand and supply from each source are likewise provided. Solve the following problem to get the best possible solution.

Sources/ Destination	**D1**	**D2**	**D3**	**D4**	**Supply**
S1	3	1	7	4	300
S2	2	6	5	9	400
S3	8	3	3	2	500
Demand	250	350	400	200	1200

TABLE 13: 3×4 TP 06

Solutions of the TP 06 are described in Table 14 by proposed and existing discussed methods.

3 Results and Discussion

The proposed method involves the calculation of variance between two selected cells for the rows and columns respectively as penalty. It is evident that the returned solution 1060 is optimum by proposed MVOTPST method from TP 01, whereas VAM, LCM and NWCM return 1100, 1115, and 1120 respectively, for the identical problem. Various examples have been tested by the developed technique and compared with

Transportation Problem 02: Consider a (3×4) TP [10]

Sources/ Destination	D1	D2	D3	D4	Supply
S1	4	5	8	4	52
S2	6	2	8	1	57
S3	8	7	9	10	54
Demand	60	45	8	50	163

Transportation Problem 03: Consider a (4×5) TP [6]

Sources/ Destination	D1	D2	D3	D4	D5	Supply
S1	4	4	9	8	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	340

Transportation Problem 04: Consider a (3×4) TP [8]

Sources/ Destination	D1	D2	D3	D4	Supply
S1	20	22	17	4	120
S2	24	37	9	7	70
S3	32	37	20	15	50
Demand	60	40	30	110	240

Transportation Problem 05: Consider a (3×4) TP [7]

Sources/ Destination	D1	D2	D3	D4	D5	Supply
S1	7	6	4	5	9	40
S2	8	5	6	7	8	30
S3	6	8	9	6	5	20
S4	5	7	7	8	6	10
Demand	30	30	15	20	5	100

TABLE 12: Transportation problems

Proposed MVOTPST Method	NWCM Method	LCM Method	VAM Method
2900	4400	2950	2850

TABLE 14: Solutions of (3×4) TP 06

the existing methods, the suggested technique has been shown to be superior to the other existing methods at both IBFS and optimal solutions. Table 3, illustrates the TPs 1-6 by various discussed methods and the proposed technique is found to be better than existing methods in the terms of IBFS.

3.1 Graphical Representation of the Results

Figure 1-6 give the graphical representation of the obtained results.

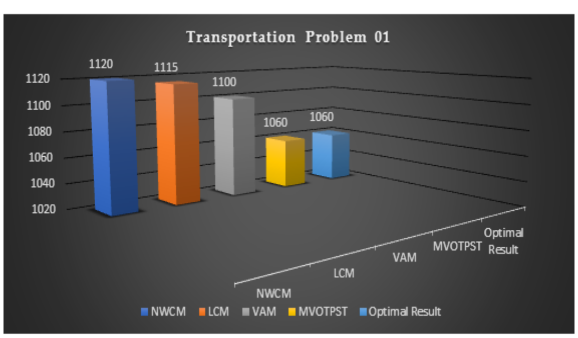

Fig. 1: Optimal Solution: Utpal Kanti Das et al. [7]

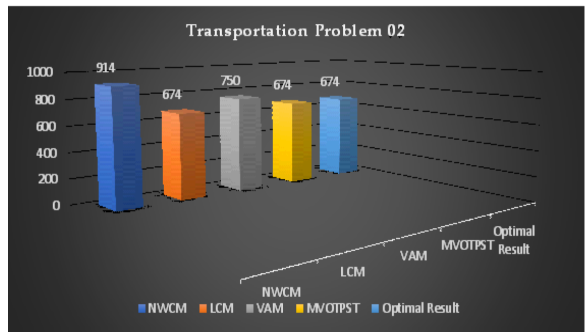

Fig. 2: Optimal Solution: M.Wali Ullah et al. [11]

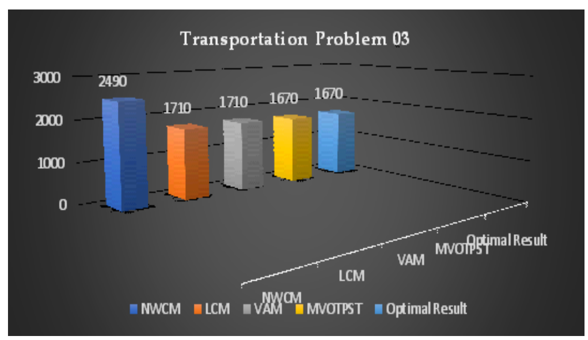

Fig. 3: Optimal Solution: Utpal Kanti Das et al. [7]

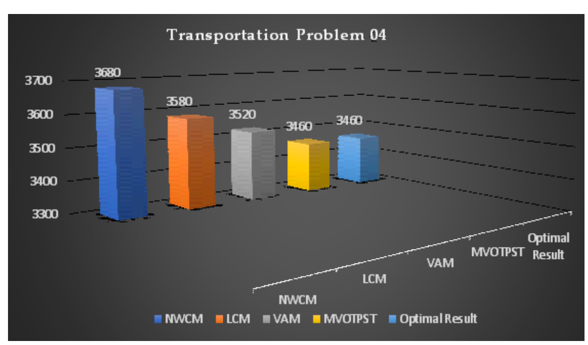

Fig. 4: Optimal Solution: Muhammad Junaid et al. [9]

TP	MVOTPST	NWCM	LCM	VAM	Remarks
1 [6]	1060	1120	1115	1100	Optimal Solutions of the proposed MVOTPST method have been observed.
2 [10]	674	914	674	750	
3 [6]	1670	2490	1710	1710	
4 [8]	3460	3680	3580	3520	
5 [7]	510	635	510	510	
6 [20]	2900	4400	2950	2850	

TABLE 15: Comparison of the proposed method with the exiting methods

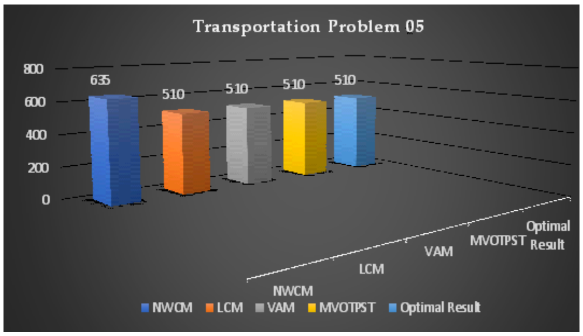

Fig. 5: Optimal Solution: Md. Ashraful Babu et al. [8]

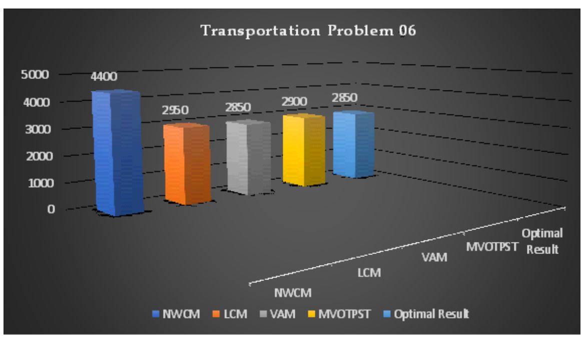

Fig. 6: Optimal Solution: Using MODI Method

4 Conclusions

Although VAM is considered to be significantly more efficient than the other approaches, there are some cases when these two methods (NWCM and LCM) outperform VAM. Researchers have provided their formulated strategies to minimize the shipping costs and have compared their techniques to the results of VAM. The proposed method MVOTPST is a Statistical approach to modify VAM by calculating the variance of cells by taking the two smallest costs column-wise and two largest costs row-wise as penalty. Several problems were examined, and the findings are displayed in the table above. Subsequently, the presented VAM could be a better alternative for locating IBFS of the TP, which is exceptionally near or equal to optimality than existing strategies.

References

[1] Nagraj balakrishnan, “Modified Vogel’s Approximation for solving unbalanced Transportation Problem”, Appl.Math.lett. Vol. 3, No.2, pp.9-11, 1990.

[2] Serdar korukoglu and Serkan Balli, “An improved Vogel’s Approximation Method for the Transportation problem” Mathematics and Computational Applications. Vol.16, No.2.pp. 370-381. 2011.

[3] Mohammad Kamrul Hasan, “Direct Methods for finding Optimal Solution of a Transportation Problem are not always reliable”, International Refereed Journal of Engineering and Science (IRJES), Vol. 1, No. 2. (2012).

[4] Md. Ashraful Babu, Md. Abu Helal, Mohammad Sazzad Hassan, Utpal Kanti Das, “Lowest Allocation Method”, International Journal of Scientific and Engineering Research, Volume 4, No. 11, 2013.

[5] Utpal Kanti Das, Md. Ashraful Babu, Aminur Rahman Khan, Md. Abu Helal, Dr.Md Sharif Uddin, ”Logical Development of Vogel’s Approximation Method (LD-VAM), International Journal of Scientific and technology Research, Vol.3, No. 2, 2014.

[6] Utpal Kanti Das, Md. Ashraful Babu, Aminur Rahman Khan, Dr.Md Sharif Uddin, ”Advanced Vogel’s Approximation Method (AVAM): A new approach to determine penalty cost for better feasible solution of Transportation Problem” , International Journal of Engineering Research and Technology (IJERT), Vol.3, No. 1, 2014.

[7] Md. Ashraful Babu, Md. Abu Helal, Mohammad Sazzad Hassan, Utpal Kanti Das, “Implied Cost Method”, Global Journal of Science Frontier Research: Mathematics and Decision Sciences, Vol 14, No. 1, 2014.

[8] Abdul Sattar Soomro, Muhammad Junaid, Gurudeo Anand Tularam, “Modified Vogel’s Approximation method for solving Transportation problem”, Mathematical Theory and Modeling, Vol 5, No.4. 2015.

[9] Smita Sood, Keerti Jain, “The maximum difference method to find initial basic feasible solution for transportation problem”, Asian Journal of Management Sciences, Vol.7, 2015.

[10] M.Wali Ullah, M.Alhaz Uddin, Rijwana Kawser, ”A Modified Vogel’s Aproximation Method for obtaining a good Primal Solutioin of Transportation Problem”, Annals of Pure And Applied Mathematics, Vol.11, No. 1, pp.63-71, 2016.

[11] M.M Ahmed, Aminur Rehman Khan, Faruque Ahmed, Md Sharif Uddin, “Customized Vogel’s Approximation Method (CVAM) for solving transportation Problems, Buletinul Institutulu Politehnic, Vol. 62, No. 66, pp. 31-44, 2016.

[12] Samuel, P. Raja “Algorithmic approach to unbalanced fuzzy transportation problem”, International Journal of Pure and Applied Mathematics, Vol. 113, 2017.

[13] S.M. Abdul Kalam Azad, Md. Bilal Hossain, “A new method for solving Transportation Problems considering average penalty”, IOSR Journal of Mathematics (IOSR-JM), Volume 13, No. 1, 2017.

[14] Surjit Paul, "A novel initial basic feasible solution method for transportation problem" International Journal of Advanced Research in Computer Science, Vol. 9, No. 1, 2018.

[15] Sanaullah Jamali, Muhammad Mujtaba Shaikh, Abdul Sattar Soomro, "Overview of Optimality of New Direct Optimal Methods for Transportation Problems", Asian Research Journal of Mathematics, pp. 1-10, 2019.

[16] Zahid Ali, Sayed Feroze Shah, Fozia Shaikh, "Minimizing the cost in Balanced transportation Problem by using Different Means", International Journal of Current Engineering and Technology, Vol.10, No.5, 2020.

[17] Abdul Majeed Khoso, Asif Ali Shaikh, Abdul Sami Qureshi, "Modified LCM's Approximation Method for solving Transportation Problem", Journal of Information Engineering and Applications. Vol.10, No.3, 2020.

[18] H A Hussein, M A K Shiker, "A modification to Vogel's Approximation Method to solve Transportation problems", Journal of Physics: Conference Series, Vol. 1591. No. 1. IOP Publishing, 2020.

[19] Kenan Karagul, Yusuf Sahin, "A novel approximation Method to obtain Initial Basic feasible Solution of Transportation Problem", Journal of King Saud University – Engineering Sciences, Vol. 32, 2020.

[20] Internet Source: https://www.coursehero.com/tutors-problems/Operations-Management/25936657

[21] Maitlo, A. A., Sandilo, S. H., Shaikh, A. H., Malookani, R. A., Qureshi, S. "On aspects of viscous damping for an axially transporting string." Science International, Vol. 28, No. 4, pp. 3721-3727, 2016.